NONDESTRUCTIVE ULTRASONIC CHARACTERIZATION
OF TWO-PHASE MATERIALS

FINAL REPORT

GRANT NCC1-88

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ABSTRACT

This final report describes the accomplishments obtained under
Grant NCC1-88, to develop ultrasonic methods for the nondestructive
characterization of mechanical properties of two-phase engineering
materials. The primary goal of the study is to establish relationships
between the nonlinearity parameter and the percentage of solid solution
phase in two-phase systems such as heat treatable aluminum alloys. The
acoustoelastic constant is also to be measured on these alloys for
comparison and confirmation. A major advantage, however, of the
nonlinearity parameter over that of the acoustoelastic constant is that
it may be determined without the application of stress on the material,
which makes the method more applicable for in-service nondestructive
characterization.

The results obtained on the heat treatable 7075 and the work
hardenable 5086 and 5456 aluminum alloys show that both the
acoustoelastic constant and the acoustic nonlinearity parameter change
considerably with the volume fraction of second phase precipitates in
these aluminum alloys. The nonlinearity parameter, however, is found to
be more sensitive to changes in the second phase than the acoustoelastic
constant. The results also show that these two parameters are
insensitive to changes in particle sizes of the second phase.

A mathematical model has also been developed to relate the
effective acoustic nonlinearity parameter to volume fraction of second

phase precipitates in an alloy. Although the relationship is in general nonlinear, the equation is approximated to within experimental error by a linear expression for volume fractions up to approximately 10 percent. The results are in agreement with the experimental measurements obtained on the aluminum alloy 7075.

INTRODUCTION

Since most mechanical properties are characteristics of the bulk of the solid, ultrasonic methods seem to offer the best promising techniques for these measurements. The majority of current efforts in nondestructive materials characterization are directed towards measurements of ultrasonic velocity[1-3] and/or attenuation[4,5]. Both measurements, however, suffer from severe difficulties which limit their applications[6]. Only in the case of ideal solids such as single crystals, may velocity and attenuation be related to mechanical properties[7]. Further, these measurements will be successful in the laboratory environment and not in a field environment. Attention is therefore directed towards measurements other than those of ultrasonic velocity and attenuation coefficient to be used for the nondestructive characterization of mechanical properties and residual stresses. Two parameters, namely the stress dependence of ultrasonic velocity and the nonlinearity parameter of second harmonic generation, are believed to offer the best promise for these measurements. The following gives a brief review of these two parameters.

STRESS DEPENDENCE OF ULTRASONIC VELOCITY

Calculations have shown that changes in ultrasonic velocities are linear functions of applied stress where the slope of this linear relationship provides a measure for third-order elastic constants[8]. Unknown stresses can be determined when both the velocity in the absence of stress as well as values of third-order elastic constants are known independently. The measured velocity in engineering materials, however, strongly depends on microstructural features which makes it necessary to

develop a calibration between velocity and stress for each material in order for the method to be used in the determination of unknown stresses. In addition, development of preferred orientations (texture) during deformation or fatigue, severely modify the third-order elastic-constants to be used for the calibration[9]. Efforts are underway at present to find solutions for these problems in order to use velocity measurements to determine residual stresses[10,11].

On the other hand, recent investigations on plain carbon steels have shown that acoustoelastic constants depend on the carbon content in these steels[12,13]. The results show a linear relationship between the acousto-elastic constant and the nominal precentage of ferrite phase in these steels. The results also indicate that the addition of heavy alloying elements to the steel does not change the value of the acoustoelastic constant as long as the amount of ferrite phase remains the same. This behavior indicates a strong relationship between the variations of ultrasonic velocity with stress and the amount of solid-solution phase in the alloy. If this behavior prevails in other types of steels, it would then be possible to use the acoustoelastic constant to measure the percentage of ferrite phase in steels, which control some of the mechanical properties such as strength, hardness and ductility.

This behavior has also been observed in four aluminum alloys containing a wide variety of strengthening alloying elements[14]. The acoustoelastic constants are measured on specimens made of the four aluminum alloys 6061-T4, 6061-T6, 3003-T251 and 2024-T351, and the data shows a linear dependence between acoustoelastic constant and the percentage of solid-solution phase in the alloy. This dependence is

similar to that observed in steel and confirms the conclusion drawn using the steel data.

It has to be realized, of course, that the acoustoelastic constant may depend on other variables such as microstructure and work hardening, and the results obtained are limited to specimens of the alloys investigated. Nevertheless, the results indicate a strong dependence of acoustoelastic constant on the amount of ferrite phase in the case of steels and the amount of the solid solution phase in the case of aluminum. The study is certainly a step forward towards the possibility of using this quantity in materials characterization. In both alloying series, the presence of the solid-solution phase strongly influences mechanical properties.

ACOUSTIC NONLINEARITY PARAMETER

An alternative method to the uniaxial stress measurements to determine the anharmonic behavior of a solid is the measurement of harmonic distortion of an initially sinusoidal ultrasonic wave[15,16,17]. A major advantage of this method is that the nonlinearity parameter β which describes the anharmonic behavior of the solid may be determined without the application of stress which makes the method extremely practical as a nondestructive evaluation technique. The method also can be readily adopted for measurements at temperatures other than that of room temperature. In this method, one excites a finite amplitude of longitudinal ultrasonic wave which propagates through the specimen. By measuring the fundamental amplitude and the generated second harmonic amplitude, one can determine nonlinearity parameter which constains a linear combination of the second- and the third-order elastic constants.

This can experimentally be obtained from the measurements of the absolute values of the amplitudes of the fundamental and the generated second harmonic of an initially sinusoidal longitudinal wave.

For a cubic crystal structure, the nonlinearity parameter β is the negative of the ratio of the nonlinear term to the linear term in the nonlinear wave equation for finite amplitude longitudinal waves propagation along a principal direction of the medium. For pure mode longitudinal sound waves propagating along a principal axis, the wave equation can be written in the form,

$$\rho_0 \frac{\partial^2 u}{\partial t^2} = K_2 \frac{\partial^2 u}{\partial a^2} + (3K_2 + K_3) \frac{\partial u}{\partial a} \cdot \frac{\partial^2 u}{\partial a^2} \qquad (1)$$

where K_2 and K_3 are the linear combinations of the second- and the third-order elastic constants.

Considering an initially sinusoidal distribution at a = 0, the solution of eq. (1) is of the form,

$$u = A_1 \sin(ka - wt) + (A_1^2 k^2 a/8) \ \beta \cos 2(ka - wt) + \ldots \qquad (2)$$

where a is the propagation distance, $k = 2\pi/\lambda$ is the propagation constant, and A_1 is the amplitude of the fundamental wave. The amplitude of the generated second harmonic is then given by,

$$A_2 = (A_1^2 k^2 a/8) \ \beta \qquad (3)$$

where

$$\beta = -(3K_2 + K_3)/K_2 \qquad (4)$$

and also can be written in terms of the measured quantities as

$$\beta = 8(A_2/A_1^2)(1/k^2 a) \qquad (5)$$

The nonlinearity parameter β can then be experimentally determined by measuring the absolute values of the amplitudes of the fundamental and the generated second harmonic wave signals A_1 and A_2, respectively. The development of a capacitive detector and its calibration[18] permit the absolute determination of the amplitude of finite sinusoidal ultrasonic waves.

PROGRAM OBJECTIVES

The primary goal of this research program is to establish relationships between the acoustic nonlinearity parameter of second-harmonic generation and the percentage of solid solution phase in a two-phase alloy system such as those of aluminum. Similar relationships are also to be established between acoustoelastic constants and percentage of second phase precipitates in the same alloy system. These alloys are available commercially and their mechanical properties are well documented. The acoustoelastic constants are also be measured on these alloys for comparison and confirmation. Specifically, the objectives of the study are:

1. To measure the nonlinearity parameter in aluminum alloys of the series 5XXX and 7XXX. The first series are non-heat treatable and their strength can be increased by strain or work hardening. The 7XXX series is age-hardenable, forgeable alloys used for elevated-temperature use. The main alloying element in this series is Zn in about 5-7% 12% and 1-3% of each Mg and Cu.

2. Ultrasonic velocity vs. stress on the same specimens to be used in the determination of the second-harmonic generation nonlinearity parameter. The experiments are to be performed when the specimens are subjected to stress applied in a direction perpendicular to that of ultrasonic propagation.

EXPERIMENTAL

The specimens used in this study were made of the heat treatable aluminum alloy 7075 and the work hardenable alloys 5086-H111 and 5456-H111. The nominal compositions of these alloys are shown in Table 1. Four specimens of Al 7075 and one of each Al-5456 and Al-5086 were investigated.

The work hardenable alloys were tested in the as received condition without any further heat treatment. The Al-7075 specimens were subjected to different heat treatments in order to achieve varying amounts of second phase in them. The specimens were first solution treated at 465° C for a period of 2 1/2 hours. Specimen #1 was then quenched at 0° C and allowed to warm slowly to room temperature. Specimen #2 was quenched in water at 25° C. Specimen #3 was quenched in boiling water and was then transferred to a furnace heated to 100° C and allowed to cool slowly to room temperature. Specimen #4 was solution

treated at 465° C and then quenched in water at 250 C. The specimen was then allowed to age at room temperature and the nonlinearity parameter, β, was mesured as a function of aging time. Speciment #4 was further aged at 120° C for various periods of time and the acoustoelastic constant was measured at each time.

The specimens for measuring the nonlinearity parameter were cylindrical in shape with a diameter of approximately 2.5 cm and a length of about 5 cm. After heat treatments the opposite faces of the specimens were made parallel to better than 5 μm. The end faces were then polished and hand lapped until they became optically flat and scratch free to be suitable for nonlinearity measurements.

Table 1. Nominal Composition of Aluminum Alloys

Alloy	Si	Cu	Zn	Mn	Mg	Cr	Al
5086(W)	-	-	-	0.45	4.0	0.15	rem
5456(W)	-	-	-	0.8	5.1	0.12	rem
7075(H)	-	1.6	5.6	-	2.5	0.23	rem

(H) Heat treatable

(W) Work hardenable

The method used in the determination of the nonlinearity parameter, β, is described in detail elsewhere[17]. It is calculated from the ratio of the amplitude of the harmonic signal, A_2, to the square of the fundamental signal, A_1, and using the relationship

$$\beta = \frac{8A_2}{A_1^2 k^2 a} \tag{1}$$

where k is the wave vector and a is the length of the specimen. After the nonlinearity parameter was measured, two parallel faces were machined on the specimens to form a rectangular cross section of approximately 1.7 to 2.5 cm. Each side was machined flat and opposite sides were made parallel to within 0.025 mm. The acoustoelastic constant is determined by measuring the changes in the natural sound velocity when the specimen is subjected to an external compressive stress. The pulse-echo overlap system, described in detail elsewhere[19], is used for measuring the natural sound velocity. A 10 MHz pulsed, longitudinal waves were used so that a direct comparison can be made with the harmonic generation results.

In order to determine the volume fraction of the second phase precipitates, the specimens were polished using Alumina powder and etched with NaOH solution. Micrographs were taken and the area of the second phase was determined. The volume perecentage of second phase present in the specimen was then calculated.

RESUSLTS AND DISCUSSIONS

Acoustoelastic Constant

The AEC of the work hardenable aluminum alloys, Al-5086 and Al-5456, and the heat treatable aluminum alloy Al-7075 specimens are plotted against the volume fraction of the second phase content in fig. 1. Also included in the figure are the values of the AEC for the alloys Al-2024, Al-6061, and Al-7075 in the as received condition reported by Li et al.[20]. From this figure, one can see that the work hardenable alloys, Al5086 and Al-5456, do not show any significant changes in the AEC due to changes in the volume fraction of the second phase. The

change in the second phase content in the two alloys is 71% while the AEC changes by only 3.4%. These results do not agree with the measurements of Schneider et al [21] who report that the AECs of the work hardenable aluminum alloys 5052, 3003 and 1100 increase linearly with the increase in the volume fraction of second phase up to about 8%.

The AECs of the heat treatable alloys, Al-2024, Al-6061 and Al-7075 as received, reported by Li et al[20] are found to increase with the increasing amounts of second phase. As the second phase content is incresed further the AECs are found to be insensitive to changes in the second phase content. Li et al report a change of 59% in the AECs for a change in second phase from 0.3% to 2.9%. The AEC of the Al-7075 alloy increases by 9% when subjected to heat treatments which result in increased amounts of second phase. The second phase content is increased from 2.9% to 6.3% for speciment #1, 7.7% for specimen #2 and 9.7% for specimen #3. The AEC of the three specimens, however, remains unchanged within the experimental error estimated for the determination of this quantity.

Salama et al[22] predicted a linear relationship between the change in the AEC and the volume fraction of second phase. In his calculatins, he considered dilute solutions of rigid particles in a matrix and found that the change in the AEC is a linear function of the concentration of second phase which agree with the experimental behavior of AEC at low concentrations. As the concentration increases the dilute solution approximation becomes invalid and the AEC is no longer sensitive to changes in the volume fraction of the second phase at higher volume fractions (fig. 1).

The results obtained on specimen #4 for the AEC as a function of aging time are plotted in fig. 2 which shows that the AEC does not change significantly with aging time, though the average size of the precipitate particleals is expected to change. It appears to indicate that the AEC is not significantly influenced by changes in the size and distribution of the second phase particles.

Nonlinearity Parameter

Fig. 3 shows the nonlinearity parameter as a function of the volume fraction of the second phase in the aluminum alloys used in this investigation. Also included in the figure are the results of Li et al[20] obtained on the aluminum alloys 2024, 6061 and 7075 in the as received condition. From this data it is seen that the nonlinerity parameter increases with the increase of the volume fraction of the second phase and that there is a linear relatinship between the nonlinearity parameter and the volume fraction. The nonlinearity parameter changes from 5.1 to 13.8% for a change in the volume fraction from 0.3 to 9.7%. The aluminum alloys 2024, 6061 and 7075 have different alloying elements and form different precipitates. The effects of these precipitates on the nonlinearity parameter, however, are the same for the same volume fraction inspite of the vast differences in their chemical compositions and properties.

The work hardenable alloys Al-5086 and Al-5456 show a different behavior where the nonlinearity parameter decreases with the increasing concentration of second phase precipitates. Schneider et al.[21] measured the AEC of the work hardenable aluminum alloys Al-1100, Al-3003 and Al-5052 and found that it increased with the increase in the volume fraction of second phase. The AEC and the nonlinearity parameter are

both functions of the second and third order elastic constants, however, one can see that they are affected differently by the presence of second phase. Fig. 4 shows the nonlinearity parameter as a function of aging time. Again no significant change in β is observed on aging up to 237 hours. This shows that β is insensitive to changes in the precipitate size and depends only on the voilume fraction of second phase.

In order to understand the behavior of the nonlinearity parameter β shown in Figs. 3 and 4, a mathematical model has recently been developed by Cantrell et al. to relate this parameter to the total volume fraction of second phase precipitates. A detailed description of this model and the mathematical derivations are shown in the paper included in Appendix A. Although the developed relationship is in general nonlinear, it can be approximated to within experimental error by a linear expression for volume fractions up to approximately 10%. This result is in agreement with those obtained experimentally and shown in Fig. 3. Finally it was assumed in the model that the number of randomly oriented grains contained within a pathlength of the propagating sound wave is sufficiently large to provide a good statistical smapling of quasi-isotropic behavior. The results shown in Fig. 4 indicate no significant variation in β although the average size of the precipitates is expected to increase, which supports the assumption of the mathematical model.

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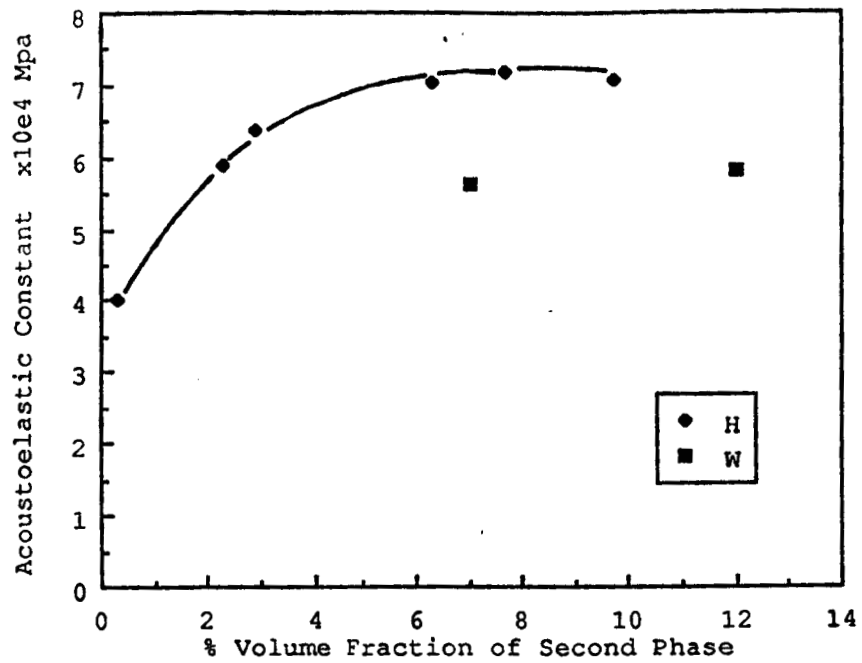   NDE 5,   , 1985.

Fig. 1 Characteristics of the Acoustoelastic
Constant for the Heat Treatable(H) and
Work Hardenable(W) Al Alloys as a Function
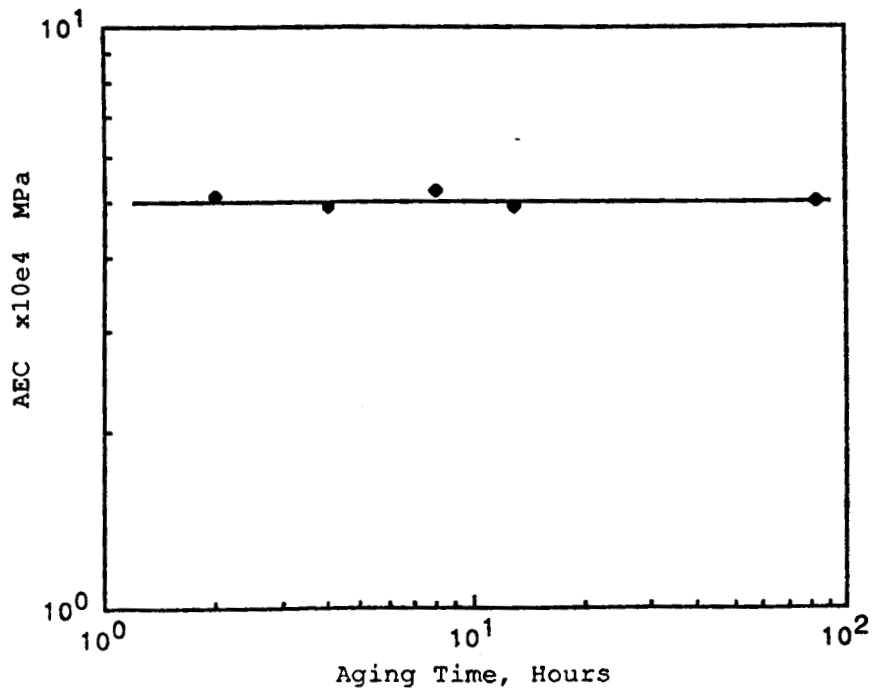of the Volume Fraction of Second Phase.

Fig. 2 Acoustoelastic Constant of the Aluminum
Alloy Al-7075 Aged at 120°C after
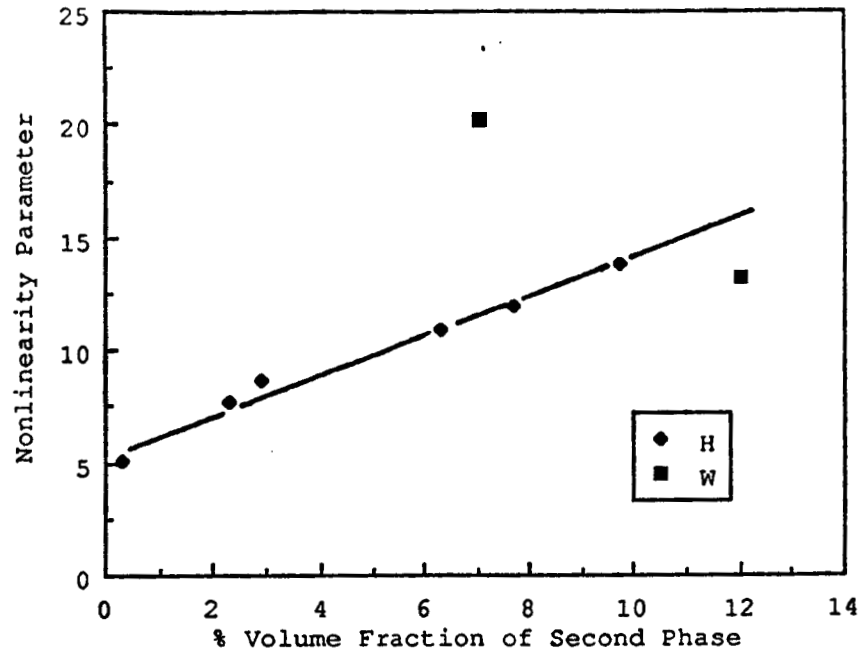Solution Treatment as a Function
of Aging Time

Fig. 3 Nonlinearity Parameter as a Function of
the Volume Fraction of Second Phase for
the Heat Treatable(H) and Work Hardenable
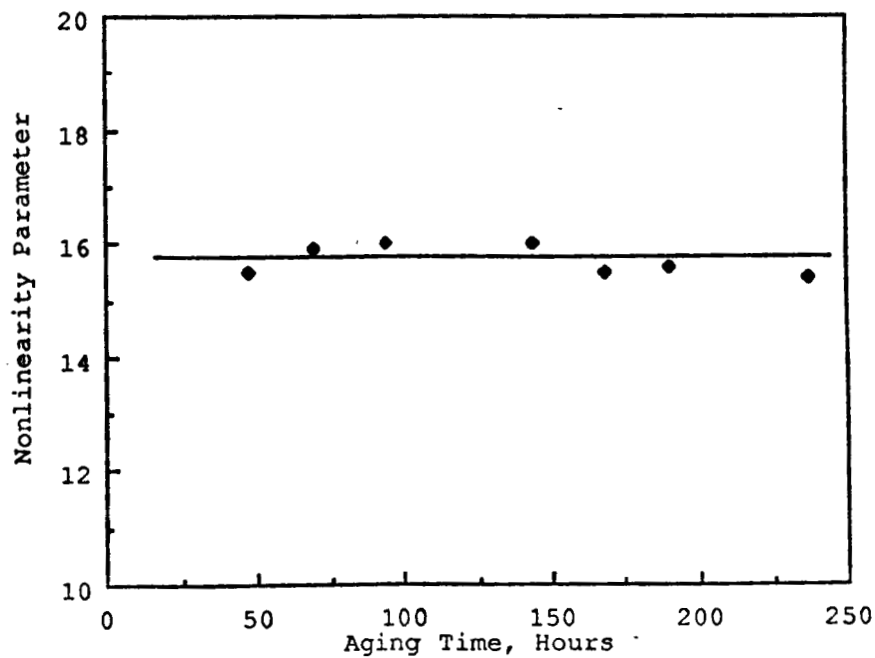(W) Al Alloys. Also included are results
reported by Li et al (1985).

Fig. 4 Change in the Nonlinearity Parameter
of the Al-7075 Specimen with Aging
Time at 25°C

APPENDIX A

EFFECTIVE NONLINEARITY PARAMETERS OF ALUMINUM ALLOYS AS A FUNCTION
OF VOLUME FRACTION OF SECOND PHASE PRECIPITATES

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ABSTRACT

A mathematical model is presented giving the
effective acoustic nonlinearity parameter of an
alloy as a function of total volume fraction of
second phase precipitates. Although the
relationship is in general nonlinear, the equation
is approximated to within experimental error by a
linear expression for volume fractions up to
approximately 10 percent. The results are in
agreement with experimental measurements of
aluminum alloy 7075 using the harmonic generation
technique.

I. Introduction

The acoustic nonlinearity parameter is a
quantitative measure of intrinsic anharmonicity in
materials that is related directly to the shape of
the interatomic potential. Such anharmonicity is
responsible for a variety of important phenomena
including harmonic generation, acoustic radiation
stresses, and the stress dependence of the sound
velocity, as well as thermal expansion, the
variation in the optical refractive index with
temperature and pressure, and the temperature
dependence of the elastic constants. Recent
studies have revealed that the magnitude of the
nonlinearity parameters along a given crystalline
direction is highly ordered according to the type
of crystalline structure[1]. This fact, together
with the establishment of modal acoustic radiation
stresses in crystalline solids,[2,3] has led to
equations expressing the thermal expansion
coefficient[4] and the temperature dependence of the
elastic constants[5] directly in terms of the modal
nonlinearity parameters.

The significant role played by the nonlinearity
parameters in determining the thermoelastic
properties of crystals leads us now to explore the
effect of microstructure on the nonlinearity
parameters. In particular, the mechanical
properties of many engineering materials are
derived, at least in part, from the presence of
secondary phases in the solid solution matrix. The
presence of the second phase, for example, raises
the flow stress; and the extent of strengthening
depends to first order on the volume fraction,
size, and characteristics of the second phase
precipitates which form during the manufacturing
process. The purpose of this paper is to present a
mathematical model of the effective nonlinearity
parameter of a quasi-isotropic solid in terms of
the volume fraction of its second phase

precipitates. We conclude with an experimental
test of the model for aluminum alloy 7075 having
volume fractions of second phase precipitates up to
ten percent.

II. Mathematical Model

For a wave of polarization $j = 1, 2, 3$ propagating
along direction q in a crystalline solid the
general relationship between the stress
field τ_{vq} and the displacement gradient $\partial P_j/\partial a_l$
(a_l is the Lagrangian coordinate) may be written to
first order in the nonlinearity as[2]

$$\tau_{vq} = \mu_j^{vq} \frac{\partial P_j}{\partial a_l} + \frac{1}{2} \nu_{jj}^{vq} \left(\frac{\partial P_j}{\partial a_l}\right)^2 \qquad (1)$$

where μ_j^{vq} and ν_{jj}^{vq} are linear combinations of second

and third order elastic constants. We shall
restrict our considerations to compressional
($j = 1$) waves in quasi-isotropic solids (i. e.
solids consisting of randomly oriented grains). We
thus drop the subscripts and write for
compressional stresses

$$\tau = \mu (\partial P/\partial a) - \frac{1}{2} \mu\beta (\partial P/\partial a)^2 \qquad (2)$$

where β is the acoustic nonlinearity parameter.
Solving for $(\partial P/\partial a)$ in terms of τ we obtain

$$\partial P/\partial a = \frac{1}{\mu} \tau + \frac{1}{2} \frac{\beta}{\mu^2} \tau^2. \qquad (3)$$

We now consider the solid to consist of any number
of phases N. We assume that for a given phase i
the grain orientations are perfectly random (no
texture) and that the number of such grains
contained within a pathlength of sound is
sufficiently large to provide a good statistical
sampling (i. e., no statistical bias). To the
extent such conditions are maintained the value of
the nonlinearity parameter is expected to be
independent of grain size.

In order to obtain the appropriate mixing law for β
we begin by defining V_o and ρ_o to be the initial
(unperturbed) volume and mass density,
respectively, of the solid. The local
transformation from the initial state to the
deformed state V or ρ is defined through the
Jacobian

$$J = \frac{V}{V_o} = \frac{\rho_o}{\rho} . \tag{4}$$

We consider that the volume at any time consists of a number N of constituent phases i such that

$$V = \sum_{i=1}^{N} V^i . \tag{5}$$

We then write from Eqs. (4) and (5)

$$J = \frac{1}{V_o} \sum_i V_i = \frac{1}{V_o} \sum_i J_i V_o^i$$

$$= \sum_i J_i f_i \tag{6}$$

where $J_i = V^i/V_o^i$ is the Jacobian for phase i and $f_i = V_o^i/V_o$ is the volume fraction of phase i.

Expanding the Jacobian in terms of the displacement gradients $(\partial u_i/\partial a_j) = u_{ij}$ and keeping the linear terms (small strains), we write (Einstein summation)

$$J \cong 1 + U_{kk}. \tag{7}$$

Substituting Eq. (7) into Eq. (6) we get

$$U_{kk} = \sum_i U_{\ell\ell}^i f_i. \tag{8}$$

We now consider the quasi-isotropic solid and assume that for a given phase i the crystal grain orientations are sufficiently random and of sufficiently large number that each phase responds individually as an isotropic structure. Under such conditions, Eq. (8) may be written

$$\frac{\partial P}{\partial a} = \sum_i (\partial P/\partial a)_i f_i \tag{9}$$

in the notation of Eq. (2).

From Eqs. (3) and (9) we obtain

$$\frac{1}{\mu}\tau + \frac{1}{2}\frac{\beta}{\mu^2}\tau^2 = (\sum_i \frac{1}{\mu_i} f_i)\tau + (\sum_i \frac{1}{2}\frac{\beta_i}{\mu_i^2} f_i)\tau^2 \tag{10}$$

where we assume local equality of stresses throughout the solid. Equating like powers of τ we obtain

$$\frac{1}{\mu} = \sum_i \frac{1}{\mu_i} f_i \tag{11}$$

and

$$\beta = \mu^2 \sum_i \frac{\beta_i}{\mu_i^2} f_i. \tag{12}$$

We thus find in general a nonlinear relationship between the effective nonlinearity parameter β and the volume fraction f_i of individual phases because of the appearance of μ^2 in Eq. (12). It is of interest to point out that for liquid media the second-order elastic constants ($C_{44} = o$ and $C_{11} = C_{12}$. In this case, $\frac{1}{\mu_i} = K_i$ in Eq. (11) where K_i are liquid state compressibilities and Eq. (12) becomes identical to the results of Apfel[6] for immiscible liquid mixtures.

We now assume that the solid consists of any number of distinct second phase precipitates and that the relative volume fractions of constituent second phase precipitates are constant. Hence, the effective nonlinearity parameter β_p and the effective μ_p of the second phase precipitates taken collectively are also constant. The invariance of the relative volume fractions of second phase precipitates must necessarily come at the expense of the solid solution constituents. We shall assume that the depletion of solid solution occurs linearly as

$$f_i = \overline{f}_i - e_i f_p \tag{13}$$

where f_i is the present volume fraction of solid solution constituent i, f_p is the total volume fraction of second phase precipitates, $\overline{f}_i$ is the volume fraction of constituent i in pure solid solution (i. e., when $f_p = o$), and e_i is the "depletion" constant for constituent i. It is of interest to note that $\Sigma\overline{f}_i = 1$ since the total volume fraction of pure solid solution ($f_p = o$) must be unity. Hence, from Eq. (13) we obtain

$$\Sigma f_i + f_p \Sigma e_i = 1 \tag{14}$$

where the sums are over solid solution constituents only. It follows from Eq. (14) that $\Sigma e_i = 1$ since the constituent volume fractions of solid solution and the total volume fraction of second phase precipitates ($f_p \neq o$) must also sum to unity.

We now write Eq. (11) as

$$\frac{1}{\mu} = \sum_i \frac{1}{\mu_i} f_i + \frac{1}{\mu_p} f_p \tag{15}$$

where in Eq. (15) and in all following equations the Σ denotes summation over solid solution constituents only. From Eqs. (13) and (15) we may write

$$\frac{1}{\mu} = \frac{1}{\mu} + f_p \left(\frac{1}{\mu_p} - \sum_i \frac{1}{\mu_i} e_i\right) \tag{16}$$

where

$$\frac{1}{\bar{\mu}} = \sum_i \frac{1}{\mu_i} \bar{f}_i. \qquad (17)$$

is the pure solid solution μ^{-1}. Similarly, from Eqs. (12), (13), and (16) we find that the effective nonlinearity parameter β of the solid in terms of total volume fraction of second phase precipitates f_p is given by

$$\beta = [1 + f_p(\frac{\bar{\mu}}{\mu_p} - \sum_i \frac{\bar{\mu}}{\mu_i} e_i)]^{-2}$$

$$x \ [\bar{\beta} + f_p(\frac{\bar{\mu}^2}{\mu_p^2}\beta_p - \sum_i \frac{\bar{\mu}^2}{\mu_i^2}\beta_i e_i)] \qquad (18)$$

where

$$\bar{\beta} = \bar{\mu}^2 \sum_i \frac{\beta_i}{\mu_i^2} \bar{f}_i \qquad (19)$$

is the nonlinearity parameter for pure solid solution. For typical values of the μ's and β's the coefficients of the terms containing f_p in Eq. (18) are estimated to be of order unity. Expanding Eq. (18) in a power series for small values of f_p and keeping only the linear terms we obtain

$$\beta = \bar{\beta} (1 + Kf_p) \qquad (20)$$

where the constant

$$K = \sum_i \frac{\bar{\mu}}{\mu_i} (2 - \frac{\beta_i}{\bar{\beta}} \frac{\bar{\mu}}{\mu_i})e_i - \frac{\bar{\mu}}{\mu_p} (2 - \frac{\beta_p}{\bar{\beta}} \frac{\bar{\mu}}{\mu_p}). \qquad (21)$$

Equation (20) is a linear approximation to Eq. (18); a survey of typical values of μ_i and β_i indicates that the equation should be accurate for most materials to within the experimental uncertainty for volume fractions as high as 10 percent.

III. Experiments

We now consider experimental confirmation of Eq. (20) for measurements of β in the heat treatable aluminum alloy 7075. Four specimens of Al 7075 were subjected to different heat treatments in order to achieve varying amounts of second phase in them. The specimens were first solution treated at 465°C for a period of 2 1/2 hours. Specimen 1 was then quenched at 0°C and allowed to warm slowly to room temperature. Specimen 2 was quenched in water at 25°C. Specimen 3 was quenched in boiling water and was then transferred to a furnace heated to 100°C and allowed to cool slowly to room temperature. Specimen 4 was solution treated at 465° and then quenched in water at 250°. The specimen was then allowed to age at room temperature and the nonlinearity parameter was measured as a function of aging time.

All specimens were cylindrical in shape with a diameter of approximately 2.5 cm and a length of about 5 cm. The opposite faces of the specimens were lapped parallel to within 41 arc seconds. The end faces were polished to optical flatness. The method used in the determination of the nonlinearity parameter is described elsewhere[7]. The measured nonlinearity parameters were corrected for the effects of attenuation[8].

In order to determine the volume fraction of the second phase precipitates, the specimens were polished using Alumina powder and etched with NaOH solution. Micrographs were taken and the area of the second phase was determined. The volume percentage of second phase present in the specimen was then calculated.

IV. Results and Discussion

The results of the experimental measurements in Al 7075 of the effective nonlinearity parameter β as a function of volume fraction of second phase precipitates f_p is shown in fig. 1. A least squares fit to the data results in the linear curve given by the solid line. The correlation coefficient of the curve is 1.0 which indicates perfect agreement between the linear approximation of Eq. (20) and the experimental data. Measurements of the intercept and slope of the curve yield a value of 6.29 for the nonlinearity parameter $\bar{\beta}$ of pure solid solution of Al 7075 and a value of 0.12 for K.
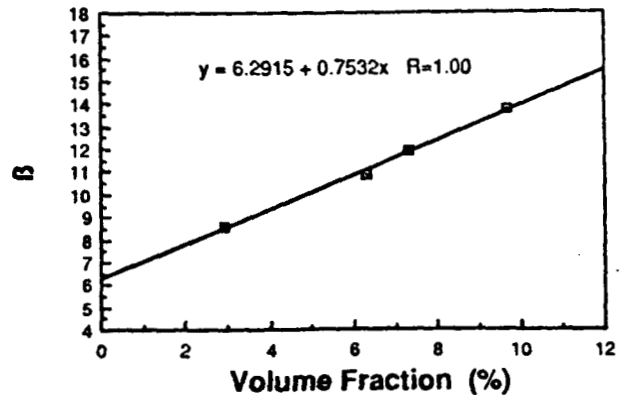
β vs. Volume % of Second Phase

Fig. 1 Effective nonlinearity parameter of Al 7075 as a function of volume fraction of second phase. Solid curve is from theory; points are from experiments.

Finally, it was assumed in the mathematical model that the number of randomly oriented grains contained within a pathlength of the propagating sound wave is sufficiently large to provide a good statistical sampling of quasi-isotropic behavior. It is expected that the 5 cm pathlength in the present specimens allows a wide variation of average grain size without violating the quasi-isotropic assumption. A manifestation of proper statistical sampling would be the invariance of the nonlinearity parameter as a function of grain size. Figure 2 shows the measured nonlinearity parameters

in Al 7075, corrected for attenuation, as a
function of aging time up to 237 hours. We observe
no significant variation in β although the average
size of the precipitates is expected to change. We
infer from these results and from fig. 1 that the
present mathematical model qualitatively predicts
the correct variation in the effective nonlinearity
parameter of Al 7075 as a function of volume
fraction of second phase precipitates. However, a
more comprehensive systematic experimental study is
necessary to quantitatively establish the validity
of the present model. Such a study is in progress.
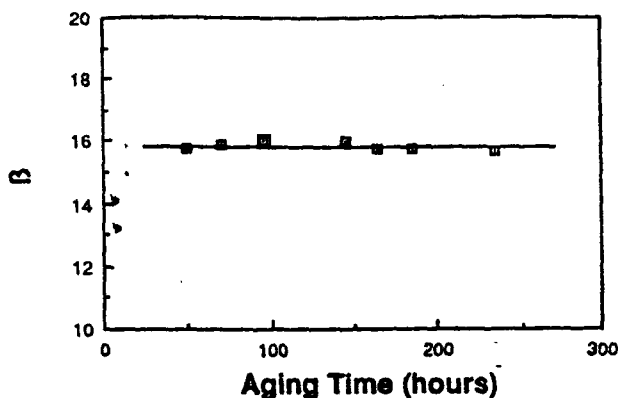
β vs aging time, Al-7075

Fig. 2 Nonlinearity parameter of Al 7075 as
function of aging time at 25°C.

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